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Towards the large N limit of pure $\mathcal{N} = 1$ Super Yang Mills

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Abstract

We find the gravity solution corresponding to a large number of NS or D fivebranes wrapped on a two sphere so that we have pure $\mathcal{N} = 1$ super Yang-Mills in the IR. The supergravity solution is smooth, it shows confinement and it breaks the $U(1)_R$ chiral symmetry in the appropriate way. When the gravity approximation is valid the masses of glueballs are comparable to the masses of Kaluza Klein states on the fivebrane, but if we could quantize strings on this background it looks like we should be able to decouple the KK states.

1 Introduction

The AdS/CFT correspondence [1, 2, 3] gives the large N dual description for $\mathcal{N} = 4$ super Yang Mills. It would be nice to find similar correspondences for pure Yang-Mills theories with less supersymmetry. By "pure" we mean without matter. In this paper we make some progress in this direction by finding a geometry that is dual to a little string theory that reduces to pure $\mathcal{N} = 1$ super Yang-Mills in the IR. We consider a little string theory [4], or NS 5 brane theory, in type IIB string theory. In the IR this theory reduces to six dimensional super Yang Mills with sixteen supercharges. We wrap this brane on S^2 and we twist the normal bundle in such a way that we preserve only 1/4 of the supersymmetries and we give a mass to the four scalar fields. This theory reduces then to pure $\mathcal{N} = 1$ super Yang-Mills in the IR. Starting from the geometric description of the NS 5 brane theory, we modify the boundary conditions for the metric to take into account that we are wrapping an S^2 and we have an appropriately twisted normal bundle. Then we find the supergravity solution using methods similar to the one in [5], i.e. reducing the problem to gauged supergravity in seven dimensions, reading the solution from [6] (see also [7], for further studies on the solution) and then lifting it up to ten dimensions using [8, 9]. When the supergravity approximation is valid the little string theory scale and the scale of the four dimensional theory are comparable. Nevertheless the solution has all the expected qualitative features. It has a $U(1)_R$ symmetry broken in the UV to Z_{2N} and the full solution breaks it further to Z_2 and we find N different solutions. The theory is confining and it is magnetically screening. It has domain walls between the different vacua. Strings can end on the domain walls.

When we try to take the decoupling limit we find a rather precise RR σ model that we should quantize in order to find the decoupled string theory describing $\mathcal{N} = 1$ super Yang-Mills, though some aspects are a bit unclear at this stage. It is not clear, for example, how the string coupling is quantized.

While this paper was in preparation we became aware of the work of I. Klebanov and M. Strassler [10] which has some overlap with ours since they also study a theory that reduces to pure $\mathcal{N} = 1$ Yang-Mills in the IR. We also became aware of a paper by C. Vafa on the same topic that appeared after the first version of this work [23]. We thank them for discussions and for telling us about the relevance of the deformed conifold metric.

2 NS 5-branes on S^2

Since the appropriate UV description of the fivebrane theory is the little string theory, or NS-5brane, we start with an NS 5-brane in type IIB string theory. The geometry

dual to the little string theory is

$$\begin{aligned} ds^2_{str} =& dx_6^2 + N(d\rho^2 + d\Omega_3^2) \\ e^{\phi} =& e^{\phi_0 - \rho} \end{aligned} \tag{1}$$

where ϕ_0 is an arbitrary constant that can be changed by shifting ρ. N is the number of fivebranes. This represents a fivebrane whose worldvolume is R^6. Now we would like to consider a brane whose worldvolume is

$$ds_6^2 = dx_4^2 + Ne^{2g} d\Omega_2^2 \tag{2}$$

so that the brane is wrapped on a two sphere of radius $R^2 = Ne^{2g}$. The factor of N is introduced just for later convenience. In order to preserve supersymmetry we should twist the normal bundle. This twisting is achieved by embedding the spin connection into the R-symmetry group. Since the non-trivial part of the spin connection is in a $U(1)$ subgroup we should choose how to embedd the $U(1)$ in $SO(4) \sim SU(2)_R \times SU(2)_L$. $SO(4)$ is the R-symmetry group of the NS fivebrane and it rotates the S^3 in (1). If we embedd the spin connection in $U(1)_R \subset SU(2)_R \subset SO(4)$ we preserve only four supercharges or $\mathcal{N} = 1$ supersymmetry in four dimensions. Let us see this more precisely from the fivebrane worldvolume point of view. The spinors that generate the supersymmetries on the NS fivebrane are two six dimensional spinors with positive chirality that are in the $(\mathbf{2}, \mathbf{0})$ of $SU(2)_R \times SU(2)_L$ and two negative chirality spinors in the $(\mathbf{0}, \mathbf{2})$. The supersymmetries that are generated by the spinors transforming under $SU(2)_L$ are broken. The preserved supersymmetries have positive chirality in six dimensions and are such that the $U(1)_R$ charge is correlated with the chirality of the spinor in the two directions of the sphere. We see that this leaves us with 1/4 of the original supersymmetries of the five-brane. The four scalars transverse to the fivebrane transform under the $(\mathbf{2}, \mathbf{2})$ of $SU(2)_R \times SU(2)_L$. This implies that, after twisting, they become spinors on the two sphere so that they do not have any zero modes. In the IR the only massless fields are the gauge fields and the gauginos. So in the IR we have pure $\mathcal{N} = 1$ super Yang-Mills. The value of the Yang-Mills coupling is given in terms of the volume of the sphere by

$$\frac{1}{g_4^2} = \frac{Vol_{S^2}}{g_6^2} = \frac{Ne^{2g}}{2\pi^2} \ . \tag{3}$$

We have described this in terms of the low energy field theory on the fivebrane so we are implicitly assuming that the volume of the S^2 is much larger than the five dimensional gauge coupling so that we can apply the low energy description for the fields on the fivebrane. We will later discuss more precisely the limit in which the four dimensional super Yang-Mills theory is expected to decouple. For the moment we will analyze this fivebrane theory, without taking the decoupling limit. This twisted fivebrane theory

seems to have a $U(1)_R$ symmetry which is the $U(1)_R$ that we are twisting. This is the $U(1)_R$ symmetry of $\mathcal{N} = 1$ super Yang-Mills, it acts on the gluinos but not on the gauge fields. We will see that this $U(1)_R$ symmetry is broken to Z_{2N} by worldsheet instantons in the NS description. This twisting also preserves the $SU(2)_L$ symmetry of the fivebrane theory. But this symmetry does not act on the massless fields, it only acts on the Kaluza Klein modes which are expected to decouple in the IR.

3 Finding the gravity solution

As explained in [5] we need to impose an appropriate boundary condition for the geometry. In this case the boundary is at $\rho \to \infty$. So we impose the condition that the seven dimensional geometry has a boundary which is $R^4 \times S^2$ and we implement the twisting by imposing appropriate boundary conditions for the seven dimensional gauge fields which come from the isometries of S^3. In this case we will impose that the $U(1)_R$ gauge field in $SU(2)_R$ is equal to the spin connection on the S^2.In other words, we set $A^3 = \cos\theta d\varphi$ for large ρ. It turns out that an ansatz like this is possible only if we allow the volume of S^2 to grow as $\rho \to \infty$. This is related to the running of the coupling in four dimensions. The string frame geometry will involve only the six dimensions parametrized by ρ, the two-sphere and the coordinates on the three sphere. In fact this solution is a particular case of the general solutions analyzed in [11]. It is a "compactification" with torsion to four dimensions. It is not really a compactification because the four dimensional Newton's constant is zero in our case since we are decoupling gravity[1]. Though the general conditions for a supersymmetric compactification were completely spelled out in [11] it is convenient, in order to find an explicit solution, to use a different technique.

Since the boundary conditions are imposed on seven dimensional fields it is convenient to work with seven dimensional gauged supergravity [12], which is a truncation to seven dimensions of the ten dimensional equations in the near horizon region of a five brane [8, 9]. This seven dimensional theory contains the metric, a dilaton, $SU(2)_R$ gauge fields and a $B_{\mu\nu}$ field, which accounts for the seven dimensional components of the B field. In [12] the seven dimensional $B_{\mu\nu}$ was dualized into a three form with a four form field strength. We set this field to zero for the time being. From the discussion above we expect that in string frame this seven dimensional solution will be R^4 times a three dimensional geometry parametrized by ρ and the coordinates on S^2, with non-zero components of the gauge fields only along these three dimensions. We can see this explicitly from the form of the supersymmetry variations in seven dimensional

[1]In [11] the heterotic case was considered, but the results extend simply to the type II case.

string frame

$$\begin{aligned}\delta\lambda &= \not{D}\phi\epsilon - \frac{i\sqrt{N}}{4}\Gamma^{\mu\nu}F_{\mu\nu}\epsilon + \frac{1}{\sqrt{N}}\epsilon \\ \delta\chi_\mu &= (D_\mu + iA_\mu)\epsilon - \frac{i\sqrt{N}}{2}F_{\mu\nu}\Gamma^\nu\epsilon\end{aligned} \tag{4}$$

The gauge fields are

$$A_\mu = \frac{1}{2}\sigma^a A^a_\mu \ , \qquad F = \frac{1}{2}F^a\sigma^a \ , \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \epsilon^{abc}A^b_\mu A^c_\nu \tag{5}$$

where σ^a are the Pauli matrices and the spinors carry an extra $SU(2)_R$ index on which these Pauli matrices act. Let us first try a simple ansatz which correctely describes the UV (or large ρ) part of the solution

$$\begin{aligned}ds^2_{str} =& dx_4^2 + N[d\rho^2 + e^{2g(\rho)}(d\theta^2 + \sin^2\theta d\varphi^2)] \\ A^3 =& \cos\theta d\varphi\end{aligned} \tag{6}$$

and all other gauge fields equal to zero. We see that this ansatz respects the boundary conditions that we want to impose at infinity. Putting this in the supersymmetry equations (4) we find the solution

$$e^{2g(\rho)} = \rho \ , \qquad \phi = \phi_0 - \rho + \frac{1}{4}\log\rho \tag{7}$$

We have absorbed the integration constant in the definition of ϕ_0. The dependence of g on ρ is related to the dependence of the four dimensional coupling (3) on the scale ρ.

This metric (6) is singular at $\rho = 0$ and the singularity is of a bad type according to the criteria in [13, 5]. The necessary ingredient in order to resolve the singularity comes in when we consider the symmetries of this solutions. The metric we found still has the $U(1)_R$ symmetry, these are $U(1)$ charge rotations in the σ^3 directions in this seven dimensional description. We expect, however that this symmetry should be broken by the choice of vacuum in the four dimensional gauge theory. Naively we expect that the solution should be such that the S^2 should shrink to zero. A similar effect (actually, the opposite) was found in the topological string theory/Chern Simons correspondence in [14] where one starts with a large N Chern Simons theory on S^3, where the S^3 is that of a resolved conifold, and one ends with a dual geometry which is a conifold resolved to a finite size S^2 [2]. In our case we cannot shrink the S^2 to zero because there is a non-trivial $U(1)$ flux through it. If we view this bundle as an $SU(2)$ bundle then it becomes clear that we can first trivialize the bundle and then shrink the S^2 to zero. Actually, this problem is completely analogous to a magnetic monopole in $SU(2)$ theory vs. the Dirac monopole. The solutions we found here is analogous to

[2] This point was emphasized to us by C. Vafa.

a Dirac monopole [6][7]. So we should look for the solution analogous to the $SU(2)$ monopole, which will have A^1, A^2 non-vanishing. These fields are charged under $U(1)_R$ and will thus break the $U(1)$ symmetry. Fortunately this solution was found in [6]. Actually, the solution in [6] is a monopole-like solution of a four dimensional gauged supergravity. In order to see that it is the same as the solution we want we should write the supersymmetry variation equations of [6] in string frame. Their solution only involves three of the dimensions and the susy equations are the same as ours in those three dimensions. So we can simply read off their solution [6]

$$\begin{aligned} A =& \frac{1}{2}\left[\sigma^1 a(\rho) d\theta + \sigma^2 a(\rho)\sin\theta d\varphi + \sigma^3 \cos\theta d\varphi\right] \\ a(\rho) =& \frac{2\rho}{\sinh 2\rho} \\ e^{2g} =& \rho\coth 2\rho - \frac{\rho^2}{\sinh^2 2\rho} - \frac{1}{4} \\ e^{2\phi} =& e^{2\phi_0}\frac{2e^g}{\sinh 2\rho} \end{aligned} \tag{8}$$

We see that for large ρ these functions go as $e^{2g} \sim \rho$, $a \sim o(e^{-2\rho})$ and the dilaton also has the same behaviour as in the previous $U(1)$ solution. This implies that the solution has the proper UV behaviour. At the origin $\rho = 0$ the metric goes as $e^{2g} \sim \rho^2$ so that the metric is non-singular. It is also easy to check that A is pure gauge at the origin. In fact this can be seen through the following gauge transformation

$$h = e^{\frac{i\sigma^1\theta}{2}} e^{\frac{i\sigma^3\varphi}{2}} \ , \qquad iA = dhh^{-1} + o(\rho^2) \ . \tag{9}$$

Note that in this monopole-like solution there is no Higgs field.

Now that we have found the seven dimensional solution it is possible to lift it up to ten dimensions using the formulas in [8, 9]. In order to write the solution it is useful to choose Euler angles on the sphere S^3, and define the left invariant one forms by viewing the sphere as the SU(2) group

$$\begin{aligned} g =& e^{\frac{i\psi\sigma^3}{2}} e^{\frac{i\tilde{\theta}\sigma^1}{2}} e^{\frac{i\phi\sigma^3}{2}} \\ \frac{i}{2} w^a\sigma^a =& dgg^{-1} \\ w^1 + iw^2 =& e^{-i\psi}(d\tilde{\theta} + i\sin\tilde{\theta} d\phi) \ , \qquad w^3 = d\psi + cos\tilde{\theta} d\phi \end{aligned} \tag{10}$$

Using the uplifting formulas in [8, 9] the ten dimensional solution is

$$\begin{aligned}
ds^2_{str} =& dx_4^2 + N\left[d\rho^2 + e^{2g(\rho)}(d\theta^2 + \sin^2\theta d\varphi^2) + \frac{1}{4}\sum_a (w^a - A^a)^2\right] \\
e^{2\phi} =& e^{2\phi_0}\frac{2e^{g(\rho)}}{\sinh 2\rho} \\
H^{NS} =& N\left[-\frac{1}{4}(w^1 - A^1)\wedge(w^2 - A^2)\wedge(w^3 - A^3) + \frac{1}{4}\sum_a F^a \wedge (w^a - A^a)\right]
\end{aligned} \tag{11}$$

The only integration constant in the solution is ϕ_0, which is the value of the dilaton at $\rho = 0$. We see that geometrically the resolution of the singularity is the same as that in [10]. If we wrap branes on the S^2 of a resolved conifold the twisted field theory on the brane is precisely what we had above and the resolution is that the S^2 shrinks and the S^3 stays with finite size. In fact our solution is similar to the solution considered in [10] except that we have only fractional branes and no regular branes.

4 The fate of the $U(1)$ R-symmetry and the N vacua

Let us understand why the $U(1)$ symmetry of the solution at infinity is broken to Z_{2N}. In the coordinates we have chosen this $U(1)$ symmetry corresponds to shifting $\psi \to \psi + \epsilon$, with $\psi = \psi + 4\pi$. This symmetry is broken by worldsheet instantons. Naively we would say that instantons of the field theory, which are the strings of the little string theory, are string worldsheets wrapping S^2. This is almost right except that the instanton also wraps an S^2 inside S^3. More precisely, if we parametrize it by the coordinates $\theta,\ \varphi$ of (11) we also we have to set $\tilde{\theta} = \theta,\ \phi = \varphi,\ \psi = const.$ It is possible to have a worldsheet with constant ψ thanks to the gauge field A^3 since what appears in the metric is $d\psi + \cos\tilde{\theta}d\phi - \cos\theta d\varphi$. In other words, the coordinate ψ is trivially fibered over the worldsheet so so that we can pick a configuration with constant ψ. There will be a flux of the B field over this sphere. This flux however depends on the point ψ where the sphere is sitting. In fact we can see that, for large ρ,

$$\frac{1}{2\pi}\int_{\psi_2} B - \frac{1}{2\pi}\int_{\psi_1} B = \frac{1}{2\pi}\int H d\psi d\theta d\varphi = -N(\psi_2 - \psi_1) \tag{12}$$

So this flux goes as

$$\frac{1}{2\pi}\int_{\psi} B = b - N\psi \tag{13}$$

This flux is the phase that appears in the worldsheet instanton calculation. This should be identified with the phase that appears in the field theory instanton calculation, which is the field theory θ_{FT} angle. We see here that, as we perform a shift in ψ, the phase changes. This implies that the $U(1)$ symmetry is anomalous, it is changing the field

theory since it is changing the θ_{FT} angle. By convention we can define the field theory theta angle to be the flux at $\psi = 0$ and then agree not to change it by perfoming $U(1)$ transformations. We see, however, that θ_{FT} is not changed if we do rotations by $\psi \to \psi + \frac{2\pi n}{N}$, with $0 \leq n < 2N$. This is precisely the surviving Z_{2N} symmetry in the UV. This symmetry is broken to Z_2 by the solution (11) . The surviving Z_2 is just $\psi \to \psi + 2\pi$ which does not change the solution (11) .

We should now explain why we have precisely N solutions, or N vacua, for each value of θ_{FT}. First we notice that the worldsheet that we were talking about around (12) is contractible in the full geometry. In order to see this we can bring the sphere close to $\rho = 0$ in the geometry (11) and then perform the gauge transformation (9), which amounts to a coordinate transformation on the three sphere. After this, the worldsheet is wrapped on the two sphere that collapses to zero. If the geometry is to be smooth the flux on the collapsing spherical worldsheet better be a multiple of 2π. This will not happen in general. For example, for the solution (11) we see that if we pick a worldsheet wrapping the sphere at $\psi = 0$ and we transport it to the origin, we do not pick any extra flux since the radial components of H in (11) projected to the worldsheet worldvolume are proportional to $\sin\psi$ and we are at $\psi = 0$. So the flux at the origin is the same as the flux at infinity which in turn is equal to θ_{FT}. So the solution (11) is a good solution only for $\theta_{FT} = 0$. Which are the other solutions?. It is easy to see how to generate new solutions. All we have to do is to rotate the gauge fields by a $U(1)$ transformation and replace the gauge fields in (11) by

$$A' = e^{\frac{i\psi_0\sigma^3}{2}} A e^{\frac{-i\psi_0\sigma^2}{2}} \tag{14}$$

This does not change the gauge fields at infinity, so it does not modify the solution in the UV. Now we can see that if we have a worldsheet wrapping near $\rho = \infty$ at the angle ψ_0 then this worldsheet can be contracted to the origin with no change in flux, since now the radial component of H projected onto the worldsheet is proportional to $\sin(\psi - \psi_0)$. But the flux of this worldsheet is $\theta_{FT} - N\psi_0$. It is this flux that should be a multiple of 2π so we see that we have N different solutions corresponding to $\psi_0 = \frac{\theta_{FT}}{N} + \frac{2\pi n}{N}$ with $0 \leq n < N$.

Let us summarize this discussion. From the purely metric point of view, all the solutions with arbitrary values of ψ_0 are non-singular, but once we consider the B fields we see that only N of the solutions are non-singular.

It is easy to see what the gravity dual of a domain wall separating two vacua is. Physicaly we expect it to be something localized near $\rho \sim 0$ since the theory is the same in the UV on both sides of the domain wall. But when we cross the domain wall we get two different solutions with different values of ψ_0 and we get a change in the flux of B over the contractible sphere by k units if $\Delta\psi_0 = \frac{2\pi k}{N}$. This implies that the domain wall should be k NS 5 branes wrapping S^3.

It is also possible to see how we can make N of those fivebranes disappear. This is easier to see from the seven dimensional point of view. We said that the seven dimensional theory in the variables of [12] has a three form potential. The fivebranes wrapped on S^3 are electrically charged under this three form potential. In the seven dimensional lagrangian there is a coupling of the form

$$iN \int A_3 \wedge Tr(F \wedge F) \tag{15}$$

where F is the field strength for the $SU(2)_R$ gauge fields. So we see that if we have N fivebranes we can replace them by an instanton of the $SU(2)$ gauge field and then expanding the instanton to infinite size we see that this kind of domain wall can disappear. This effect is of course familiar in the heterotic string context where we can transform an NS fivebrane into an instanton in the gauge group [15]. In that case one fivebrane was the same as one instanton.

5 Towards the pure $\mathcal{N} = 1$ theory

If we intend to decouple the four dimensional theory we will have to take a limit where we go to scales much lower than the little string mass scale. As shown in [16] we need to S-dualize the gravity solution and switch to a D-fivebrane description.

The S-dual metric to that in (11) is

$$\begin{aligned} ds^2_{str} =& e^{\phi_D} \left[dx_4^2 + N(d\rho^2 + e^{2g(\rho)} d\Omega_2^2 + \frac{1}{4} \sum_a (w^a - A^a)^2) \right] \\ e^{2\phi_D} =& e^{2\phi_{D,0}} \frac{\sinh 2\rho}{2e^{g(\rho)}} \end{aligned} \tag{16}$$

and the NS H field becomes a RR H field. Everything that we said in the previous section about worldsheet instantons translates into D-string instantons.

In this description an external quark is a fundamental string that comes in from infinity. When we have a quark anti-quark pair and we separate them by a large distance we see that we find a finite string tension from the point of view of the four dimensional theory equal to

$$T_s = \frac{e^{\phi_{D,0}}}{2\pi\alpha'} \tag{17}$$

The masses of glueballs and Kaluza-Klein states on the spheres is, in the supergravity approximation,

$$M^2_{glueballs} \sim M^2_{KK} \sim \frac{1}{N\alpha'} \tag{18}$$

Finally the tension of a domain wall interpolating between the nth and $n+1$th vacua, which is now a D5 brane, is

$$T_{wall} \sim N^{3/2} e^{2\phi_{D,0}} \tag{19}$$

Fundamental strings can end on these domain walls [17]. The baryon vertex is a D3 brane wrapped on S^3. A magnetic monopole source is a D-3 brane wrapping the sphere that the worldsheet instantons were wrapping in the previous section and extending in the radial and time directions. They are screened because, since the sphere is contractible, each member of a monopole anti-monopole pair can be wrapped in the three dimensional space parametrized by ρ and the contractible sphere.

We see that in order to decouple the scale of the string tension from the scale of the KK states we need $e^{\phi_{D,0}}N \ll 1$. This goes beyond the gravity approximation, which requires $e^{\phi_{D,0}}N \gg 1$, but it seems that we could still use this metric to formulate a string theory. This string theory should be such that it essentially has no excitations on S^2 or S^3. This is plausible since the sizes of those spheres is smaller than the string scale. Presumably we should be able to replace the six dimensional part of the geometry by a Liouville-like theory; since this geometry is similar to the near conifold geometry, this sounds plausible. In fact, for the near conifold geometry it was suggested that the sigma model can be replaced, for some calculations, by the $c = 1$ (super) Liouville theory. Indeed, in [18] it was proved that the CFT is a Kazama- Suzuki $SL(2)/U(1)$ coset model with level $k = 3$. This theory was studied in the context of non-critical bosonic strings in ref.[19] and the relation between bosonic strings and SCFT of a conifold agrees with the results of [20].

It would be nice to understand the mapping to a Liouville-like theory in the case that we have RR fields. A nice feature of this RR sigma model is that it seems possible to choose light cone gauge. In AdS it is hard to choose light cone gauge, because in Poincare coordinates we have a horizon. In this case there is no horizon and the light cone theory should be better defined. In the purely four dimensional theory we do not expect to have any dimensionless parameter. In our case we have a dimensionless parameter which is ϕ_0, this parameter is related to the ratio of the QCD string tension (or mass scale) and the six dimensional gauge coupling, or six dimensional scale of the little string theory. Presumably once we exchange the spheres by a Liouville theory we would find that the string coupling is fixed in the IR and of order $1/N$.

Another related point is the precise coefficient for the beta function. In the 5-brane theory it is natural to define the scale as g_{00} in D-string metric, since that will be the energy of a massive string mode sitting at position ρ. This gives a relation between the scale in the field theory and the position ρ of the form $\mu \sim e^{\rho/2}$. When we look at the definition of the four dimensional string coupling in (3) we see that $1/(g_4^2 N) \sim \log \mu/\Lambda_{QCD}$. But the coefficient is not the correct one. It is interesting that if we go to the five dimensional Einstein frame metric and we define the scale as $\mu^2 \sim g_{00}^{5,E}$ then we get precisely the right β function with the right numerical coefficient [21]. We could not find any precise reason for choosing this UV/IR relation. In order to determine the precise relation it seems that we should know the precise string theory

and sigma model.

In summary, this solution seems to provide a starting point for constructing the large N limit of pure $\mathcal{N} = 1$ Yang-Mills. We expect that the S^3 and S^2 would disappear from the sigma model, leaving only the radial direction, and probably also an angular direction, representing the $U(1)$ symmetry. The final picture would have the flavor of that in [22], but it seems crucial to have RR fields in order to generate a warp factor in string frame.

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References

[1] J. Maldacena, "The large N limit of superconformal field theories and supergravity," *Adv. Theor. Math. Phys.* **2** (1998) 231–252, `hep-th/9711200`.

[2] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, "Gauge theory correlators from non-critical string theory," *Phys. Lett.* **B428** (1998) 105, `hep-th/9802109`.

[3] E. Witten, "Anti-de Sitter space and holography," *Adv. Theor. Math. Phys.* **2** (1998) 253–291, `hep-th/9802150`.

[4] M. Berkooz, M. Rozali, and N. Seiberg, "On transverse fivebranes in M(atrix) theory on T**5," *Phys. Lett.* **B408** (1997) 105–110, `hep-th/9704089`.

[5] J. Maldacena and C. Nunez, "Supergravity description of field theories on curved manifolds and a no go theorem," `hep-th/0007018`.

[6] A. H. Chamseddine and M. S. Volkov, "Non-Abelian BPS monopoles in N = 4 gauged supergravity," *Phys. Rev. Lett.* **79** (1997) 3343–3346, `hep-th/9707176`.

[7] A. H. Chamseddine and M. S. Volkov, "Non-Abelian solitons in N = 4 gauged supergravity and leading order string theory," *Phys. Rev.* **D57**, 6242 (1998) `hep-th/9711181`.

[8] M. Cvetic, H. Lu, and C. N. Pope, "Consistent Kaluza-Klein sphere reductions," hep-th/0003286.

[9] A. H. Chamseddine and W. A. Sabra, "D = 7 SU(2) gauged supergravity from D = 10 supergravity," *Phys. Lett.* **B476** (2000) 415, hep-th/9911180.

[10] I. R. Klebanov and M. J. Strassler, "Supergravity and a confining gauge theory: Duality cascades and (chi)SB-resolution of naked singularities," hep-th/0007191.

[11] A. Strominger, "Superstrings with torsion," *Nucl. Phys.* **B274** (1986) 253.

[12] P. K. Townsend and P. van Nieuwenhuizen, "Gauged seven dimensional supergravity," *Phys. Lett.* **B125** (1983) 41.

[13] S. S. Gubser, "Curvature singularities: The good, the bad, and the naked," hep-th/0002160.

[14] R. Gopakumar and C. Vafa, "On the gauge theory/geometry correspondence," hep-th/9811131.

[15] J. Curtis G. Callan, J. A. Harvey, and A. Strominger, "World sheet approach to heterotic instantons and solitons," *Nucl. Phys.* **B359** (1991) 611.

[16] N. Itzhaki, J. M. Maldacena, J. Sonnenschein, and S. Yankielowicz, "Supergravity and the large N limit of theories with sixteen supercharges," *Phys. Rev.* **D58** (1998) 046004, hep-th/9802042.

[17] E. Witten, "Branes and the dynamics of QCD," *Nucl. Phys. Proc. Suppl.* **68** (1998) 216.

[18] D. Ghoshal and C. Vafa, "C = 1 string as the topological theory of the conifold," *Nucl. Phys.* **B453** (1995) 121–128, hep-th/9506122.

[19] S. Mukhi and C. Vafa, "Two-dimensional black hole as a topological coset model of c = 1 string theory," *Nucl. Phys.* **B407** (1993) 667 hep-th/9301083.

[20] E. Witten, "Ground ring of two-dimensional string theory," *Nucl. Phys.* **B373** (1992) 187–213, hep-th/9108004.

[21] K. Intriligator and N. Seiberg, "Lectures on supersymmetric gauge theories and electric- magnetic duality," *Nucl. Phys. Proc. Suppl.* **45BC** (1996) 1–28, hep-th/9509066.

[22] O. Aharony, M. Berkooz, D. Kutasov, and N. Seiberg, "Linear dilatons, NS5-branes and holography," *JHEP* **10** (1998) 004, `hep-th/9808149`.

[23] C. Vafa, "Superstrings and topological strings at large N," `hep-th/0008142`.